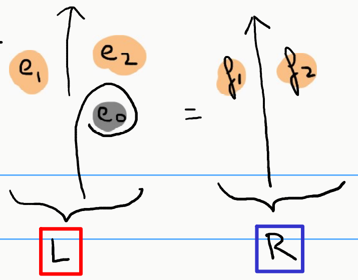
R1:



$e_0 \longmapsto e_0$

$e_i \longmapsto e_i - \langle e_i, e_0\rangle e_0 = f_i$, $i \neq 0$

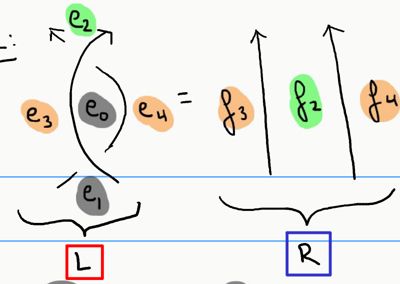
$$\begin{array}{c|c} & \begin{matrix} e_0 & e_1 & e_2 & e_3 & \end{matrix} \\ \begin{matrix} e_0 \\ e_1 \\ e_2 \\ e_3 \\ \\ \end{matrix} & \begin{bmatrix} 1 & 1 & 2u & 0 & \cdots \\ 1 & *+1 & *+2u & * & \cdots \\ 2u & *+2u & *+4u^2 & * & \cdots \\ 0 & * & * & * & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array} \longrightarrow \begin{array}{c|c} & \begin{matrix} e_0 & f_1 & f_2 & f_3 & \end{matrix} \\ \begin{matrix} e_0 \\ f_1 \\ f_2 \\ f_3 \\ \\ \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & * & * & * & \cdots \\ 0 & * & * & * & \cdots \\ 0 & * & * & * & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}$$

Matrix for $\boxed{L}$

$\sigma = 1$

Matrix for $\boxed{R}$

R2:



$e_{0,1} \longmapsto e_{0,1}$

$e_2 \longmapsto e_2 + e_1 \longmapsto e_2 + e_1 + \langle e_2 + e_1, e_1\rangle e_0 = f_2$

$e_i \longmapsto e_i + \langle e_i, e_1\rangle e_0 = f_i$, $i = 3, 4, \dots, n$

Matrix for $\boxed{L}$

$$\begin{array}{c|c} & \begin{matrix} e_0 & e_1 & e_2 & e_3 & e_4 & e_5 & \end{matrix} \\ \begin{matrix} e_0 \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ \\ \end{matrix} & \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & \cdots \\ -1 & 1-2u^2 & 0 & x_{1,3}-u & x_{1,4}-u & x_{1,5} & \cdots \\ 0 & 0 & 2u^2-1 & y_{1,3}+u & y_{1,4}+u & y_{1,5} & \cdots \\ 0 & x_{1,3}-u & y_{1,3}+u & 0 & 0 & * & \cdots \\ 0 & x_{1,4}-u & y_{1,4}+u & 0 & 0 & * & \cdots \\ 0 & x_{1,5} & y_{1,5} & * & * & * & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}$$

$e_2 \longmapsto e_2 + e_1$

$$\begin{array}{c|c} & \begin{matrix} e_0 & e_1 & e_2+e_1 & e_3 & e_4 & e_5 & \end{matrix} \\ \begin{matrix} e_0 \\ e_1 \\ e_2+e_1 \\ e_3 \\ e_4 \\ e_5 \\ \\ \end{matrix} & \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & \cdots \\ -1 & 1-2u^2 & 1-2u^2 & x_{1,3}-u & x_{1,4}-u & x_{1,5} & \cdots \\ 0 & 1-2u^2 & 0 & x_{1,3}+y_{1,3} & x_{1,4}+y_{1,4} & x_{1,5}+y_{1,5} & \cdots \\ 0 & x_{1,3}-u & x_{1,3}+y_{1,3} & 0 & 0 & * & \cdots \\ 0 & x_{1,4}-u & x_{1,4}+y_{1,4} & 0 & 0 & * & \cdots \\ 0 & x_{1,5} & x_{1,5}+y_{1,5} & * & * & * & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}$$

$\sigma = 0$

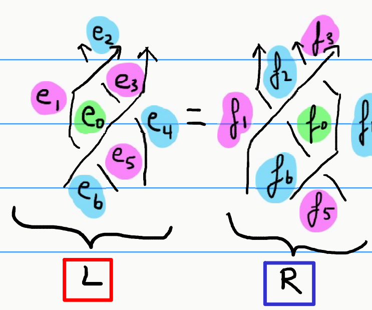
$$\begin{array}{c|c} & \begin{matrix} e_0 & e_1 & e_2+e_1+\langle e_2+e_1, e_1\rangle e_0 & e_3+\langle e_3,e_1\rangle e_0 & e_4+\langle e_4,e_1\rangle e_0 & e_5+\langle e_5,e_1\rangle e_0 & \end{matrix} \\ \begin{matrix} e_0 \\ e_1 \\ e_2+e_1+\langle e_2+e_1, e_1\rangle e_0 \\ e_3+\langle e_3,e_1\rangle e_0 \\ e_4+\langle e_4,e_1\rangle e_0 \\ e_5+\langle e_5,e_1\rangle e_0 \\ \\ \end{matrix} & \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & \cdots \\ -1 & 1-2u^2 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & x_{1,3}+y_{1,3} & x_{1,4}+y_{1,4} & x_{1,5}+y_{1,5} & \cdots \\ 0 & 0 & x_{1,3}+y_{1,3} & 0 & 0 & * & \cdots \\ 0 & 0 & x_{1,4}+y_{1,4} & 0 & 0 & * & \cdots \\ 0 & 0 & x_{1,5}+y_{1,5} & * & * & * & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}$$

matrix for $\boxed{R}$

$e_i + \langle e_i, e_1\rangle e_0 \longleftarrow e_i$, $i = 3, 4, \dots, n$

$e_2 + e_1 + \langle e_2 + e_1, e_1\rangle e_0 \longleftarrow e_2 + e_1$

R3:



$e_0 \longmapsto e_0 = f_0$

$i = 1, 2, \dots, 6$ $\begin{cases} i \text{ odd } e_i \longmapsto e_i - \frac{1}{2u+1} e_0 = f_i \\ i \text{ even } e_i \longmapsto e_i + \frac{1}{2u+1} e_0 = f_i \end{cases}$

This change of basis changes the L matrix to the R matrix

(the map is the "identity" on everything not pictured)

Matrix for $\boxed{L}$

$\langle e_0, e_0\rangle = 4u^2 - 1 = (2u-1)(2u+1)$

$i = 1, \dots, 6$ $\begin{cases} i \text{ odd } \langle e_0, e_i\rangle = 2u \\ i \text{ even } \langle e_0, e_i\rangle = 1 \end{cases}$

Matrix for $\boxed{L}$ +

$$\left[\begin{array}{ccccccc|cc} 0 & 1-2u & 2u-1 & 1-2u & 2u-1 & 1-2u & 2u-1 & 0 & \cdots \\ 1-2u & -1 & 0 & -1 & 0 & -1 & 0 & 0 & \cdots \\ 2u-1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ 1-2u & -1 & 0 & -1 & 0 & -1 & 0 & 0 & \cdots \\ 2u-1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ 1-2u & -1 & 0 & -1 & 0 & -1 & 0 & 0 & \cdots \\ 2u-1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right]$$

||

Matrix for $\boxed{R}$

i odd: $\langle e_0, e_i - \frac{1}{2u+1} e_0\rangle = \langle e_0, e_i\rangle + (1-2u)$

i even: $\langle e_0, e_i + \frac{1}{2u+1} e_0\rangle = \langle e_0, e_i\rangle + (2u-1)$

i odd, j odd: $\langle e_i - \frac{1}{2u+1} e_0, e_j - \frac{1}{2u+1} e_0\rangle = \langle e_i, e_j\rangle - \frac{2\cdot 2u}{2u+1} + \frac{2u-1}{2u+1} = \langle e_i, e_j\rangle + (-1)$

i odd, j even: $\langle e_i - \frac{1}{2u+1} e_0, e_j + \frac{1}{2u+1} e_0\rangle = \langle e_i, e_j\rangle + \frac{2u-1}{2u+1} - \frac{2u-1}{2u+1} = \langle e_i, e_j\rangle + 0$

i even, j even: $\langle e_i + \frac{1}{2u+1} e_0, e_j + \frac{1}{2u+1} e_0\rangle = \langle e_i, e_j\rangle + \frac{2}{2u+1} + \frac{2u-1}{2u+1} = \langle e_i, e_j\rangle + 1$

$i, j = 1, \dots, 6$